n-dimensional shape-from-moments problem

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The shape-from-moments problem consists in recovering a discrete shape - a *polytope* - from a finite set of its moments. In applications like medicine or geophysics, moments are computed from tomographic measurements. We focus on retrieving the vertices for convex polytopes.

Previous methods solved this problem only in the 2D-case, *i.e.* for polygons. They consider the problem in the complex plane, compute complex moments and recover the vertices based on an integration formula due to Davis. The approach we take works in any dimension. It is directly based on *directional moments.* Consider a polytope P and a vector z in $\mathbb{R}^n$, the directional moment $\mu_k(z)$ of order k for the polytope P is defined by

$$\mu_k(z) = \int_P \langle x, z \rangle^k dx \qquad \forall k \in \mathbb{N}, \tag{1}$$

where $\langle \cdot, \cdot \rangle$ is the usual scalar product in $\mathbb{R}^n$. These moments are directly computed from the measurements. The algorithm consists of two steps. In a first step, we recover the projections of the vertices on several directions. These projections are obtained as generalized eigenvalues of a pair of Hankel matrices. In a second step, we match the projections by a robust interpolation to obtain the set of vertices.

The shape-from-moments problem will be illustrated with self-created polytopes in the 2D-case and in the 3D-case. I will show simulations from the generation of data to the use of the algorithm to retrieve the vertices, going through the computation of the moments using an efficient formula and through the choice of a *reference direction.* The latter is a main key for minimizing the error made between the vertices generated at the beginning of the simulation and the vertices computed by our algorithm.