

Le faîte et la directrice. Ridge and directrix.

Jérémy Berthomieu, Vincent Cossart, Pascal Hivert, Hussein Mourtada

25 mars 2013

The ridge and the directrix have been studied for usual cones, *i.e.* algebraic spaces defined by homogeneous equations in a polynomial ring $k[X_1, \dots, X_n]$.

The directrix is the greatest linear space of translations which leave stable the cone.

The ridge is the greatest group of translations which leave stable the cone. They do not coincide in positive characteristic. Curiously, the easiest to define and to understand (the directrix) is the most difficult to compute. See for example [BHM].

Indeed, the directrix is used by every mathematician attending this conference and probably by every mathematician in the world: its equations may be seen as the smallest set of variables needed to write down equations of the cone. The ridge is much more subtle, it may be seen as the smallest set of additive polynomials needed to write down equations of the cone.

The ridge has been defined and used by Hironaka [H] in his attempts to solve the desingularization problem in positive or mixed characteristic case: it plays a crucial role in the cases where a proof is made (dimension ≤ 3).

In this conference we will stress on the fact that these ridge and directrix are defined and computable also in the quasi-homogeneous case, in fact in [G1][G2], Giraud made a theory of the ridge, which generalizes to this case. Neither Giraud nor Hironaka paid attention to an effective computation, their only motivation was the study of singularities and the desingularization problem.

This gives the opportunity to the oldest author to define a new invariant of singularities very helpful in the desingularization problem.

BIBLIOGRAPHIE

[BHM] J. BERTHOMIEU, P. HIVERT AND H. MOURTADA. Computing Hironaka's invariants: Ridge and Directrix. Arithmetic, Geometry, Cryptography and Coding Theory 2009, *Contemp. Math.*, vol. 521, pages 9-20. de AGCT-12.

[G1] GIRAUD, J.. Étude locale des singularités. Cours de 3^{ème} cycle, Pub. n° 26, Univ. d'Orsay 1972.

[G2] J. GIRAUD. Contact maximal en caractéristique positive, *Ann. Sc. ENS 4^{ème} série* **8** (1975), 201-234.

[H] H. HIRONAKA. Additive groups associated with points of a projective space, *Ann. Math.* **92** (1970), 327-334.