Real root finding of determinants of linear matrices

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In our work we are interested in the study of algebraic varieties given as the zero locus of the determinant of a matrix whose entries are linear forms with rational coefficients. Our main contribution concerns with the construction of efficient algorithms for finding real points in every connected component of this determinantal variety, starting from a geometric characterization of the problem. The resolution of this problem in a good complexity class, taking advantage of the geometric structure of the problem, is required in many scientific areas, like convex optimization and real convex algebraic geometry.

Let M be a matrix of size $k \times k$ of linear forms in n variables, with rational coefficients. We reduce the direct analysis of the real roots of the determinant firstly lifting to a variety defined by a bilinear system and after lifting a second time to a trilinear zero-dimensional system, whose degree is bounded by a function on k, n that is at most polynomial if either k or n is fixed and having a good generic asymptotic behavior.

Our algorithm works with these last two systems of polynomial equations and is based on the recursive computation of critical points of projections on generic lines, generating as output a parametrization of a finite set, whose reunion intersects every component of the determinantal variety, giving the requested result. What we proved is that, under genericity assumptions on the matrix M (that is when the coefficients of the linear forms are in general position) and on the choice of lines where the variety is projected, the algorithm is correct. Its complexity is polynomial in the aforementioned degree bounds which signficantly improves the state of the art.

As an example of a possible application, we remind that this algorithm can be used as a first step of a deeper analysis of the structure of a given LMI set, whose algebraic boundary is shaped by a determinant of a linear matrix. In this sense, during the talk we will also give some useful numerical examples.